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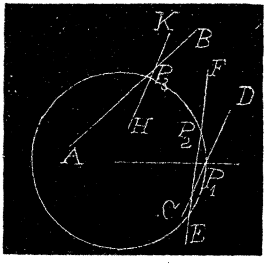
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$$P_1P_2 = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + [(dr/d\theta)]^2}\, d\theta = \int_{\theta_1}^{\theta_2} \sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\, d\theta \ldots\ldots\ldots\ldots (4),$$

which gives P_1P_2 in terms of θ_2, θ_1 being known. Then substitute from (4) in (1) to get ψ in terms of θ_2. Substitute this value, and also $f(\theta_2)$ for r_2 in (2) and (3). Then by eliminating (θ_2) we have resulting the equation to the locus of the intersecting of the lines. The solution depends on our ability to integrate (4). Now if the given lines are not straight, it is evident that the only changes are in equations (2) and (3). These may be derived from the equations to the lines in original position by a method of transformation of coördinates. For example, the equation to HK may be derived from that to AB by revolving the pole and polar axis about P_3 through an angle equal to ψ and in the opposite direction. If the given curve is a circle and the lines straight, the problem can be definitely solved as follows:



Transform coördinates so that center of circle shall be pole and OP_1 the the polar axis. Then $r=f(\theta)$ becomes $r=c$.

Let $(r_1, \theta_1)(r_2, \theta_2)(r_3, \theta_3)$ represent the new coördinates of points P_1, P_2, P_3, respectively.

Then $r_1=r_2=c$. $\theta_1=0$. $P_1P_2=c\theta_2$. From (1) $\psi=ac\theta_2/b$.

(2) becomes $r=[r_3\sin(\eta+ac\theta_2/b-\theta_3)]/\sin(\eta+ac\theta_2/b-\theta)\ldots\ldots\ldots\ldots(5)$.

(3) becomes $r=[c\sin(\eta_1+nac\theta_2/b-\theta_2)]/\sin(\eta_1+nac\theta_2/b-\theta)\ldots\ldots\ldots(6)$.

(5) can be solved for θ_2 and the result can be substituted in (6), giving the equation required. Then if desired the coördinates can be again transformed to the original form.

MECHANICS.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

43. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Missouri.

Two weights P and Q rest on the concave side of a parabola whose axis is horizontal, and are connected by a string, length l, which passes over a smooth peg at the focus, F. [*Bowser's Analytical Mechanics*, page 54.]

I. Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi; COLMAN BANCROFT, M. Sc., Professor of Mathematics, Hiram College, Hiram, Ohio; GEORGE LILLEY, LL. D., Portland, Oregon; G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas, and the PROPOSER.

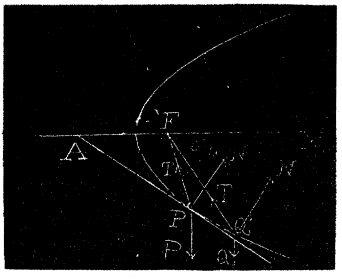
Let AX be the horizontal axis of the parabola, F the focus, P' and Q' the positions of the weights P and Q, T the tention of the string $P'FQ'$, N and N' the normal reactions at P' and Q' respectively, θ and θ' the angles between the axis AX and the focal radii to P' and Q' respectively, A the intersection of the axis and the tangent at P'. Denote the latus rectum by $4m$.

$$\angle\theta=\angle FAP'+\angle FP'A=2\angle FAP',$$

by property of parabola.

$\therefore\ \angle FAP'=\frac{1}{2}\theta.$

Since N is inclined to the vertical at the same angle that the tangent is inclined to the horizontal, we have for equilibrium of the forces at P', resolving vertically and horizontally,



$N\cos\frac{1}{2}\theta+T\sin\theta=P$, and $N\sin\frac{1}{2}\theta=T\cos\theta$,

from which

$T(\cot\frac{1}{2}\theta\cos\theta+\sin\theta)=P$, or, $T\cot\frac{1}{2}\theta=P$.

Similarly, $T\cot\frac{1}{2}\theta'=Q$. $\therefore\ \cot\frac{1}{2}\theta=P/Q\cot\frac{1}{2}\theta'$.

From the polar equation of a parabola,

$$FP'=\frac{2m}{1-\cos\theta},\quad FQ'=\frac{2m}{1-\cos\theta'}.$$

But $FP'+FQ'=l$. $\therefore\ \frac{2m}{1-\cos\theta'}=l-\frac{2m}{1-\cos\theta}$, $\cos\theta'=1-\frac{2m(1-\cos\theta)}{l(1-\cos\theta)-2m}$,

$$\sin^2\tfrac{1}{2}\theta'=\frac{m(1-\cos\theta)}{l(1-\cos\theta)-2m},\quad \cot\tfrac{1}{2}\theta'=\sqrt{\frac{l(1-\cos\theta)+m\cos\theta-3m}{m(1-\cos\theta)}}.$$

Then, $\cot\frac{1}{2}\theta=\frac{P}{Q}\sqrt{\frac{l(1-\cos\theta)+m\cos\theta-3m}{m(1-\cos\theta)}}$. Since $1-\cos\theta=\frac{2}{\cot^2\frac{1}{2}\theta+1}$, the preceding equation gives $\cot\frac{1}{2}\theta=\frac{P\sqrt{l-2m}}{\sqrt{m(P^2+Q^2)}}$.

II. Solution by F. ANDEREGG, A. M., Professor of Mathematics, Oberlin College, Oberlin, Ohio.

If, in the figure above, θ represents the angle XFP, and θ_1, XFQ, then

$FP=r=\frac{2m}{1-\cos\theta}=\frac{m}{\sin^2\frac{1}{2}\theta}$. $\therefore\ \sin^2\frac{1}{2}\theta=\frac{m}{r}$, and $\cos^2\frac{1}{2}\theta=\frac{r-m}{r}$.

Since $FQ=l-r$, $\sin^2\frac{1}{2}\theta_1=\frac{m}{l-r}$, and $\cos^2\frac{1}{2}\theta_1=\frac{l-r-m}{l-r}$.

Since the tension is the same in all parts of the string and the angle between the radius vector and tangent is half the angle between the radius vector and the X axis, $T=P\tan\frac{1}{2}\theta=Q\tan\frac{1}{2}\theta_1$.

$$\therefore \frac{P}{Q}=\frac{\operatorname{ctn}\frac{1}{2}\theta}{\operatorname{ctn}\frac{1}{2}\theta_1}, \quad \therefore \frac{P^2}{P^2+Q^2}=\frac{r-m}{l-2m}=\frac{m\operatorname{ctn}^2\frac{1}{2}\theta}{l-2m},$$

$$\therefore \operatorname{ctn}\tfrac{1}{2}\theta=\frac{P\sqrt{l-2m}}{\sqrt{m(P^2+Q^2)}}.$$

III. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio University, Athens, Ohio.

Let r', r'' be the parts of the string l joining the focus and the weights P and Q; θ and θ' the angles which r' and r'' make with the axis of X.

For the equilibrium of P and Q, resolving along the tangents through P and Q, T being the tension in the string,

$T\sin\frac{1}{2}\theta=P\cos\frac{1}{2}\theta$ ..... (1). $T\sin\frac{1}{2}\theta'=Q\cos\frac{1}{2}\theta'$ .... (2).

These give $P/Q\cot\frac{1}{2}\theta=\cot\frac{1}{2}\theta'$ .... (3).

The equations to the curve are

$$r'=\frac{2m}{1+\cos\theta}, \quad r''=\frac{2m}{1+\cos\theta'};$$

then $r'+r''=2m\left(\frac{1}{1+\cos\theta}+\frac{1}{1+\cos\theta'}\right)=l$ .... (4).

Now $\cos\theta=\frac{\cot^2\frac{1}{2}\theta-1}{\cot^2\frac{1}{2}\theta+1}$, $\cos\theta'=\frac{\cot^2\frac{1}{2}\theta'-1}{\cot^2\frac{1}{2}\theta'+1}$ .... (5).

(3) and (5) and the resulting values of $\cos\theta$ and $\cos\theta'$ in (4) and reducing gives

$$\frac{2P^2\cot^2\frac{1}{2}\theta+(P^2+Q^2)}{2P^2\cot^2\frac{1}{2}\theta}=\frac{l}{2m} \text{ .... (6).}$$

Subtracting unity from both members of (6) and taking the square root of the result,

$$\tan\tfrac{1}{2}\theta=\frac{P}{\sqrt{P^2+Q^2}}\sqrt{\frac{l-2m}{m}} \text{ ..... (7).}$$

In which put $\pi-\theta$ for θ for Bowser's result.